Using (9), (7) can be written as

$$i_{\alpha_2} = -\frac{\omega^2\mu\epsilon_1 + j\omega\mu\sigma_1}{2\gamma_s^2}$$

and (6) becomes

$$\Gamma = \gamma_s\left(1 - \frac{\omega^2\mu\epsilon_1 + j\omega\mu\sigma_1}{2\gamma_s^2}\cdot e^{-t/\tau_c}\right), \quad (10)$$

which is identical to (5).

For longer lifetimes and higher frequencies, the approximations used above are even better. In very short lifetime materials, such as Gallium Arsenide, the correction terms included in the analysis of Nag and Das[1] may become important. However, with short lifetime materials, the diffusion length becomes quite small and it is difficult to satisfy the assumption of uniform carrier density throughout the semiconductor.

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Letter to the Editor*

The following paper is felt to be of interest to workers in the microwave measurements field:

B. E. Rabinovich, "Method free from mismatching errors for measuring the loss of attenuators," *Izmeritel'naya Tekhnika*, no. 3, pp. 44–47; March, 1962.

The English translation appears in *Measurement Techniques*, no. 3, pp. 238–243; 1962.

Rabinovich places one directional coupler ahead and one after the device under test. The change in the ratio of incident power of both side arms is related to the insertion loss. The effects of finite directivity and main line VSWR are considered. By the use of auxiliary phaseshifters or sliding loads, the maximum and minimum influence of these effects can be determined.

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A Simple Relation Between Cavity Q and Maximum Rejection for Narrow-Band Microwave Band-Stop Filters*

In the course of designing band reject filters, one must often determine the unloaded cavity $Q(Q_u)$ required to achieve a predetermined maximum rejection. This situation is analogous to that of determining the Q_u required to give a specific midband loss (due to dissipation) in bandpass filters.

The design of microwave band reject filters possessing narrow stop bands has been given detailed consideration by Young, Matthaei and Jones.[1] The following relation is developed in this communication based on transformation from the low-pass filter prototype:[2]

$$L_{max} = 20\sum_{i=1}^{n} \log_{10} \omega_1' w g_i Q_{ui} + 10 \log \frac{g_0 g_{n+1}}{4} \quad (1)$$

where

L_{max} = the maximum rejection in db
w = the normalized bandwidth at half-power (Butterworth) or equiripple (Chebyshev) points
Q_{ui} = the unloaded Q of the ith cavity
n = the number of cavities
ω_1' = the normalized band edge radian frequency of the low-pass prototype
g's = the low-pass prototype element values.

If the use of equal Q_u cavities is assumed, as is most often true, (1) can be written as

$$L_{max} = 20n \log_{10}(\omega_1' w Q_u) + 20\sum_{i=1}^{n}\log_{10} g_i + 10\log_{10}\left(\frac{g_0 g_{n+1}}{4}\right). \quad (2)$$

Now ω_1' is unity by definition and w is simply the cavity decrement, the reciprocal of the loaded $Q(Q_L)$. Thus, (2) becomes

$$L_{max} = 20n \log_{10}\frac{Q_u}{Q_L} + 20\sum_{i=1}^{n}\log_{10} g_i + 10\log_{10}\left(\frac{g_0 g_{n+1}}{4}\right). \quad (3)$$

For the situation where the rejection response follows the *maximally flat* (Butterworth) characteristic, g_0 and g_{n+1} are unity. Then, the third term of (3) is −6.02 db. For n equal to 1 through 10, the second term of (3) varies between +6.00 and +6.02 db. Therefore, a good approximation for the relation of L_{max} to Q_u is

$$L_{max} = 20n \log_{10}\left(\frac{Q_u}{Q_L}\right) \text{ db} \qquad n = 1, 2, 3, \cdots, 10. \quad (4)$$

The analogous relation for midband dissipation loss in bandpass filters is[3]

$$L_{diss} = 20n \log \frac{Q_u}{Q_u - Q_L}. \quad (5)$$

A graph of (4) is shown in Fig. 1. Curves of (5) have been widely published.[4]

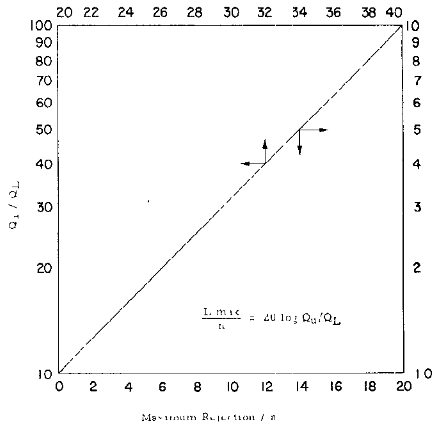

Fig. 1—Unloaded Q vs maximum rejection for Butterworth band reject filters.

Verification of (4) has been carried out by means of a simple test. A pair of UHF filters, a band reject filter and a bandpass filter, were constructed using identical resonators. The bandpass filter was a two-resonator unit. It had a 0.9-db insertion loss at f_0 = 253 Mc and a 3-db pass band of 6.7 Mc. The band reject filter was a three-resonator structure which exhibited approximately 105-db maximum rejection (measured as 35 db per resonator) at f_0 = 254 Mc with a 3-db bandwidth of 20 Mc.

Applying (4) to the band reject data, Q_L = 254/20 = 12.7 and $JL/3$ = 35 db. Thus, Q_u/Q_L = 57 and Q_u = 724.

Eq. (5) was used in the bandpass case. From the two-resonator bandpass data, Q_L = 253/6.7 = 37.86, $JL/2$ = 4.5, Q_L/Q_u = 0.0505 and, therefore, Q_u = 749. Thus, the simple experiment has shown satisfactory agreement between (4) and the widely accepted bandpass Q (5).

The range of n [in (4)] for a Butterworth response can be extended beyond 10 if one can show that the second term of (3) is approximately +6.0 db for arbitrary n. For the Butterworth response, g_i is given by

$$g_i = 2 \sin\left[\frac{(2i-1)\pi}{2n}\right]. \quad (6)$$

To satisfy the 6.0 db requirement, it must be shown that

$$\prod_{i=1}^{n} \sin\left[\frac{(2i-1)\pi}{2n}\right] = \frac{1}{2^{n-1}}. \quad (7)$$

This has been done numerically for the range of n from 1 to 10. Extension to higher values of n can be made by repeated numerical solutions of (7) using a computer. An analytic approach is to prove the identity of (7). This has been done and the result is tabulated as series No. 1049 by Jolley,[5] where n is any positive integer. On the basis of this result, (4) can be used for any value of n.

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[1] L. Young, G. L. Matthaei, and E. M. T. Jones, "Microwave band-stop filters with narrow stop bands," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. 10, pp. 416–427; November, 1962.

[2] S. B. Cohn, "Direct-coupled resonator filters," PROC. IRE, vol. 45, pp. 187–196; February, 1957.

[3] "Very High Frequency Techniques," vol. II, McGraw-Hill Book Co., New York, N. Y., p. 745; 1947.

[4] For example, "The Microwave Engineer's Handbook," 2nd ed., p. T-99; 1963.

[5] L. B. W. Jolley, *Summation of Series*, 2nd. ed. rev., Dover Publications, p. 194; 1961.